International Journal of Theoretical & Applied Sciences, **1**(2)**:** 60-70(2009) **ISSN : 0975-1718**

Rayleigh waves in a thermo-viscoleastic solid loaded with viscous fluid of varying temperature

J.N. Sharma, R. Sharma and P.K. Sharma

Department of Mathematics, NIT Hamirpur, (HP) INDIA

ABSTRACT : In the present paper is concerned with study of thermoelastic interaction in an infinite Kelvin-Voigt type viscoelastic, thermally conducting solid bordered with viscous liquid half-spaces/layers of varying temperature. Complex secular equations in closed and isolated mathematical conditions for Rayleigh wave propagation in completely separate terms are derived. The results for coupled and uncoupled theories of thermoelasticity have been obtained as particular cases from the derived secular equations. In order to illustrate and compare the theoretical results, the numerical solution is carried out for copper material by using the functional iteration method. The computer simulated results in respect of dispersion curves, attenuation coefficients, amplitudes of temperature change and displacements are presented graphically.

Keywords : Kelvin-Voigt model, Rayleigh waves, viscoelastic, attenuation

INTRODUCTION

The study of viscoelastic behavior in bone and in bio-protective materials is of interest in several contexts. Materials used for structural applications of practical interest may exhibit viscoelastic behavior which has a profound influence on the performance of that material. For example, viscoelastic shoe insoles are useful in reducing mechanical shocks transmitted to the bones and joints. Materials used in engineering applications may exhibit viscoelastic behavior as an unintentional side effect. Viscoelasticity is of interest in materials science, metallurgy, and solid state physics since it is causally linked to a variety of microphysical processes and can be used as an experimental probe of those processes. The casual links between viscoelasticity and microstructure are exploited in the use of viscoelastic tests as an inspection or diagnostic tool. Viscoelasticity was examined in the late twentieth century when synthetic polymers were engineered and used in a variety of applications.

The stress response of these materials depends on both the strain applied and the strain rate at which it was applied. Because of the time dependent material behavior of viscoelastic materials, their behavior is dependent on the history of loading and are said to have a *'memory'*. The linear theory of thermo-viscoelasticity describes the linear behavior of both elastic and anelastic materials and provides a basis for describing the attenuation of seismic waves. Before 1960, most of the work on linear viscoelastic wave propagation for which explicit solutions have been obtained was essentially two dimensional. Bland [1] has given an account of three dimensional linear viscoelasticity theory. He concluded that as in perfectly elastic isotropic medium under assumption of small displacement, two types of waves can be propagated in an isotropic viscoelastic medium when body forces are absent. The first type of wave called as dilatational or longitudinal or *P*-type wave, the second type of waves, known as shear or equivoluminal or *S*-type wave. Viscoelastic materials are of great importance in industry and therefore have attracted a lot of interest in the engineering community. On the other hand, mathematicians often find viscoelasticity interesting because of its present applications and challenges for the theory of integro-differential equations. Gurtin and Sternberg [2] discussed the linear theory of viscoelasticity.

The basic governing equations of thermoelasticity in the usual framework of linear coupled thermoelasticity consist of the wave type (hyperbolic) equations of motion and the diffusion type (parabolic) equation of heat conduction. Lord and Shulman [3], Green and Lindsay [4] and Green and Laws [5] have formulated generalized dynamical theories of thermoelasticity in which the equation of heat conduction is also hyperbolic. Some researchers such as Ackerman et al. [6], Ackerman and Overtone [7], Guyer and Krumhansl [8], proved experimentally for solid Helium that thermal waves (second sound) propagating with finite, though quite large speed, also exist although for most of the solids the corresponding frequency window namely, range of the frequency of thermal excitations in which thermal waves can be detected, is extremely limited. Green [9] proved the uniqueness of the equations derived by Green and Lindsay and studied the propagation of acceleration waves.

The existence of Rayleigh waves was predicted by Lord Rayleigh [10] in 1885 for whom they were named. They are distinct from other types of seismic waves, such as *P*-waves and *S*-waves or Love waves and travel the surface of a relatively thick solid material penetrating to a depth of one wavelength approximately. These waves combine both longitudinal and transverse motion to create an elliptic orbital motion. Rayleigh waves are useful because they are very sensitive to surface defects. These waves can be used to inspect areas that other waves might have difficulty in reaching.

Love [11] gave the first comprehensive treatment of dispersion of Rayleigh and Love waves in the case of an elastic solid half space covered by a single solid layer. Ewing et al. [12] gave a review of the works done by many authors. According to Andle and Vetelino [13], the development of micro-acoustic wave sensors in biosensing have created the need for further investigation into surface wave propagation in a fluid loaded layered medium. Kovacs et al. [14] reported a detailed experimental study on Love wave sensors for biochemical sensing in liquids. Kurtze and Bolt [15] derived a dispersion equation for bending waves when a plate is in contact with an inviscid fluid based on the acoustic impedance concept. Plona et al. [16] studied Rayleigh and lamb waves at liquid-solid boundaries. Wu and Zhu [17] studied the propagation of Lamb waves in a plate bordered with inviscid liquid layers on both sides.

Banerjee and Kundu [18] investigated ultrasonic field modeling in plates immersed in fluid. Bossart et al. [19] gave hybrid numerical and analytical solutions for acoustic boundary problems in thermo-viscous fluids. Craster and Williams [20] derived a reciprocity relation for fluid-loaded elastic plates that contain rigid defects. Wang and Zhang [21] applied boundary element method for simulating the coupled motion of a fluid and a three-dimensional body. Skelton [22] studied line force receptance of anelastic cylindrical shell with heavy exterior fluid loading. Krishna and Ganesan [23] studied the fluid filled and submerged cylindrical shells with constrained viscoelastic layer. Nayfeh and Nagy [24] derived the exact characteristic equations for leaky waves propagating along the interfaces of several systems involving isotropic elastic solids loaded with viscous fluids including semi spaces and finite thickness plates totally immersed in fluids or coated on one or on both sides by finite thickness fluid layers. The technique adopted by Nayfeh and Nagy [24] removed certain inconsistencies that unnecessarily reduced the accuracy and range of validity of Zhu and Wu [25] results. They suggested two possible models to improve the deficiency in the model used by Zhu and Wu [25]: (*i*) Modeling the viscous liquid as a hypothetical solid whose shear rigidity equals $i\omega\mu_L$, and (*ii*) use of Stokes model which splits the viscosity parameter between c_{11} and c_{13}. To study the surface waves in coated anisotropic medium loaded with viscous liquid, Wu and Wu [26] set $c_{11} = \kappa + \frac{4}{3} i\omega\mu_L$, and $c_{13} = \kappa - \frac{2}{3} i\omega\mu_L$, where $\kappa = \rho_L c_L^2$ is the bulk modulus of the liquid, where ρ_L and c_L are the density and longitudinal velocity of the viscous liquid.

Hassan and Nagy [27] investigated the leaky Rayleigh waves in a fluid filled cylindrical cavity and found that the dispersive Rayleigh wave propagating around a concave cylindrical surface is substantially less attenuated by fluid loading than the corresponding wave on a flat surface. Garadzhaev [28] gave a note on the spectral theory of problems on normal oscillation of an ideal compressible fluid in rotating elastic shell. Wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section immersed in a fluid is studied by Venkatesan and Ponnusamy [29]. Qi [30] studied the influence of viscous fluid loading on the propagation of leaky Rayleigh waves in the presence of heat conduction effects. Cho and Rose [31] investigated guided waves in a water loaded hollow cylinder. Popovics et al. [32] studied the propagation of leaky Rayleigh and Scholte waves at the fluid-solid interface subjected to transient point loading by using integral transform technique. Cherednichenko [33] studied the propagation of attenuated Rayleigh waves along a fluid-solid interface of arbitrary shape between a compressible fluid medium and an elastic solid. Mozhaev and Weihnacht [34] investigated subsonic leaky Rayleigh waves at liquid-solid interfaces.

Sharma et al. [35] studied the propagation characteristics of Rayleigh surface waves in microstretch thermoelastic continua under inviscid fluid loadings. Hasheminejad and Safari [36] studied acoustic scattering from viscoelastically coated spheres and cylinder in viscous fluids. Shuvalov et al.[37] studied the subsonic spectrum of complex velocity versus real frequency in immersed anisotropic plates is considered for the various options of presupposed choice of modes in the loading fluid half-spaces. Zhao et al. [38] utilized the finite element method to simulate the laser-excited leaky Rayleigh wave at air-solid cylindrical interfaces. Ghattas and Li [39] studied the interaction of a stationary viscous fluid with an elastic solid that undergoes large displacement. Sharma et al. [40] studied thermoelastic Lamb waves in a transversely isotropic plate bordered with layers of inviscid liquid. Sharma and Pathania [41] investigated thermoelastic waves in coated homogeneous anisotropic materials in the context of linear generalized thermoelasticity. Sharma and Pathania [41] analyzed the thermoelastic waves in a plate bordered with layers of inviscid liquid. Sharma and Pathania [42] explained crested waves in thermoelastic plates immersed in liquid. Sharma and Pathania [43] discussed the propagation of leaky surface waves in thermoelastic solids due to inviscid fluid loadings. Ugurlu et al. [44] gave a method of analysis for investigating the effects of elastic foundation and fluid on the dynamic response characteristics (natural frequencies and associated mode shapes) of rectangular Kirchhoff plates. Sorokin and Chubinskij [45] addressed free wave propagation and attenuation in elastic plates loaded by a quiescent viscous compressible fluid. Sharma and Pathania [46] investigated generalized thermoelastic Lamb waves in a plate bordered with layers of inviscid liquid or half-space of inviscid liquid on both sides in the context of generalized theories of thermoelasticity. Sharma *et. al.* [47] studied the propagation of Lamb waves in visco-thermoelastic plates under fluid loadings. Sharma and Pathania [48] investigated the propagation of generalized thermoelastic waves in anisotropic plates sandwiched between liquid layers.

BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

The constitutive relations and basic governing equations of motion and heat conduction of generalized thermo-viscoelasticity for a Kelvin-Voigt type solid :

1. Strain-Displacement relations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (i, j = 1, 2, 3) \qquad \text{...(1)}$$

2. Stress-Strain-Temperature Relations

$$\sigma_{ij} = \lambda^* \delta_{ij} e_{kk} + 2\mu^* e_{ij} - \beta^*(T + t_1 \delta_{2k} \dot{T})\delta_{ij} \quad (i, j = 1, 2, 3) \qquad \text{...(2)}$$

3. Equations of Motion

$$\sigma_{ij,j} + b_i = \rho \ddot{u}_i$$
$$q_i = -KT_{,i} \quad (i, j = 1, 2, 3) \qquad \text{...(3)}$$

4. Equation of Heat Conduction

$$KT_{,ii} - \rho C_e(\dot{T} + t_0 \ddot{T}) = \beta^* T_0(\dot{e} + t_0 \delta_{1k} \ddot{e}) - (s + t_0 \dot{s}) \quad (i, j = 1, 2, 3) \qquad \text{...(4)}$$

where

$$\lambda^* = \lambda_e\left(1+\alpha_0 \frac{\partial}{\partial t}\right), \ \mu^* = \mu_e\left(1+\alpha_1 \frac{\partial}{\partial t}\right),$$
$$\beta^* = (3\lambda^* + 2\mu^*)\,\alpha_T = \beta_e\left(1+\beta_0 \frac{\partial}{\partial t}\right)$$
$$\beta_e = (3\lambda_e + 2\mu_e)\,\alpha_T,\ \beta_0 = (3\lambda_e\,\alpha_0 + 2\mu_e\alpha_1)\,\alpha_T/\beta_e \qquad \text{...(5)}$$

Here, $\vec{u}(x, y, z, t) = (u, v, w)$ is displacement vector, K is thermal conductivity , C_e is specific heat at constant strain of the solid, ρ is the density of medium, α_0, α_1, are the viscoelastic relaxation times, $T(x, y, z, t)$ is the temperature change, e_{ij} and σ_{ij} are the strain and stress tensors, e is the dilatation, λ_e, μ_e are called Lame constants, α_T is the coefficient of linear thermal expansion, β is thermomechanical coupling, s is the heat source term, b_i are the components of body force, t_0, t_1 are the thermal relaxation times, T_0 is the initial temperature of the medium, δ_{ij} is the Kronecker's delta and $k = 1$ for *LS* theory and $k = 2$ for *GL* theory of generalized thermoelasticity. The comma notation is used for spatial derivatives and superposed dot is used for time differentiation.

The governing field equations in the context of linear generalized thermoelasticity can be written from equations 1-5 by setting mechanical relaxation parameters $\alpha_0=\alpha_1=0$. Upon taking the thermal relaxation times $t_0=0=t_1$, we obtain the governing field equations for coupled theory of thermoelasticity from equations 1-5 above. Also on setting thermal relaxation times $t_0=0=t_1$ and thermomechanical coupling $\beta=0$, the governing field equations for uncoupled thermoelasticity (elastic medium) can be obtained. The equations 1-5 are also subjected to appropriate boundary conditions consistent with the situations of various problems under investigation.

The basic governing field equations for viscous fluid (liquid) medium are given by

$$\mu_L \frac{\partial}{\partial t}\nabla^2 \vec{u}_L + \left(\lambda_L + \frac{1}{3}\mu_L \frac{\partial}{\partial t}\right)\nabla\nabla.\vec{u}_L - \beta_L^*\nabla T_L = \rho_L \frac{\partial^2 \vec{u}_L}{\partial t^2} \qquad \text{...(6)}$$

$$T_L = \frac{\beta_L^* T_0^*}{\rho_L C_V^*}\nabla.\vec{u}_L \qquad \text{...(7)}$$

where,

$\beta_L^* = 3\lambda_L\,\alpha^*$, α^* being coefficient of volume thermal expansion; λ_L is the Bulk modulus; ρ_L is the density of fluid; μ_L is the kinematic viscosity of the liquid; C_V^*, T_0^* are respectively the specific heat at constant volume and ambient temperature of fluid. Here $\vec{u}_L$ is the velocity vector and T_L is temperature deviation from the ambient temperature T_0^* of the fluid.

Boundary Conditions

The boundary conditions at the solid-liquid interface to be satisfied are as follows :

(i) The magnitude of the normal component of the stress tensor of the plate should be equal to the pressure of the liquid.

(*ii*) The tangential components of stress tensor of plate and liquid should be equal.

(*iii*) The normal component of the displacement of the plate liquid should be equal.

(*iv*) The horizontal components of displacement of both the media should also be equal at the interface because of no slip condition.

(*v*) The thermal boundary condition is given by

$$T_{,Z} + HT = 0 \qquad \text{...(8)}$$

where H is the Biot's constant. In the absence of liquid ($\rho_L = 0$, $\mu_L = 0$) that is for thermoelastic half space, the boundary condition corresponds to thermally insulated boundary for ($H \rightarrow 0$) and refers to isothermal one in case of ($H \rightarrow \infty$).

The present paper deals with the study of thermo-viscoelastic interactions in an infinite Kelvin-Voigt type viscoelastic solid bordered with viscous fluid with varying temperature. The Voigt –model of linear visco-thermo elasticity, earlier used by Kaliski [49] has been employed to consider the viscoelastic behavior of the solid. Relevant results of previous investigations, such as Graff [51], Achenbach [50] have been deduced as special cases. The generalized theory of thermo elasticity is employed to understand the effect of thermo mechanical coupling and mechanical relaxation time. Complex secular equations in closed and isolated mathematical conditions completely separate terms are derived. The results for coupled and uncoupled theories of thermoelasticity have been obtained as particular cases from the derived secular equations. In order to illustrate and compare the theoretical results in various situations, the numerical solution is carried out for copper material by employing the functional iteration method and the corresponding dispersion curves, amplitudes of displacements and temperature change are presented graphically.

FORMULATION OF THE PROBLEM

Consider a viscous liquid layer of finite thickness h overlying a homogeneous isotropic, thermally conducting viscoelastic solid in the undeformed state at uniform temperature T_0. We take origin of the coordinate system (x, y, z) at any point of the plane surface (interface) and the z-axis pointing vertically downward into the solid half-space which is thus represented by $z \geq 0$. We choose x-axis in the direction of wave propagation in such a way that all the particles on a line parallel to y-axis are equally displaced. Therefore, all the field parameters become independent of y-coordinate. Further, it is assumed that the disturbances are small and are confined to neighborhood of the interface $z = 0$ and hence vanish as $z \to \infty$.

The basic governing equations of motion (3) and heat conduction (4) for the considered solid in the context of generalized thermo-viscoelasticity in the absence of body forces and heat sources, are

$$(\lambda^* + \mu^*)\nabla(\nabla.\vec{u}) + \mu^*\nabla^2\vec{u} - \beta^*\nabla(T + t_1\,\delta_{2k}\dot{T}) = \rho\ddot{\vec{u}} \qquad ...(9)$$

$$K\nabla^2 T - \rho C_e\,(\dot{T} + t_0\ddot{T}) = \beta^* T_0\,\nabla.(\dot{\vec{u}} + t_0\,\delta_{1k}\,\ddot{\vec{u}}) \qquad ...(10)$$

where $\vec{u}(x, z, t) = (u, 0, w)$ is the displacement vector and $T(x, z, t)$ is the temperature change.

We define the quantities

$$x' = \frac{\omega^* x}{c_1},\; z' = \frac{\omega^* z}{c_1},\; T' = \frac{T}{T_0},\; \xi' = \frac{\xi c_1}{\omega^*},$$

$$u' = \frac{\rho\omega^* c_1 u}{\beta T_0},\; w' = \frac{\rho\omega^* c_1 w}{\beta_e T_0}.$$

$$\varepsilon = \frac{\beta^2 T_0}{\rho C_e(\lambda_e + 2\mu_e)},\; \omega' = \frac{\omega}{\omega^*},\; h' = \frac{\omega^* h}{c_1},$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\beta T_0},\; \delta^2 = \frac{c_2^2}{c_1^2}.$$

$$t'_1 = \omega^* t_1,\; t'_0 = \omega^* t_0,\; \alpha'_1 = \omega^*\alpha_1,$$

$$\alpha'_0 = \omega^*\alpha_0,\; t' = \omega^* t,\; c' - \frac{c}{c_1}$$

$$c_2^2 = \frac{\mu_e}{\rho},\; c_1^2 = \frac{\lambda_e + 2\mu_e}{\rho},\; \omega^* = \frac{C_e(\lambda_e + 2\mu_e)}{K}, \qquad ...(11)$$

where ω^* is the characteristic frequency of the solid plate; ε is the thermomechanical coupling constant and c_1, c_2 are respectively, the longitudinal and shear wave velocities in the thermoelastic half-space.

In view of the quantities defined in equation (11) and introducing the displacement potential functions ϕ and ψ of longitudinal and shear waves in the solid through the relations

$$u = \phi_{,x} + \psi_{,z},\; w = \phi_{,z} + \psi_{,x} \qquad ...(12)$$

in the governing equations 9-10, we obtain

$$\left(1 + \alpha_1\frac{\partial}{\partial t}\right)\nabla^2\psi = \frac{1}{\delta^2}\ddot{\psi} \qquad ...(13)$$

$$\left(1 + \delta_0\frac{\partial}{\partial t}\right)\nabla^2\phi - \left(1 + \beta_0\frac{\partial}{\partial t}\right)(T + t_1\delta_{2k}\dot{T}) = \ddot{\phi} \qquad ...(14)$$

$$\nabla^2 T - (\dot{T} + t_0\ddot{T}) = \varepsilon_T\left(1 + \beta_0\frac{\partial}{\partial t}\right)\nabla^2\left(\dot{\phi} + t_0\delta_{1k}\ddot{\phi}\right) \qquad ...(15)$$

where $\delta_0 = \alpha_0 + 2\delta^2(\alpha_1 - \alpha_0)$.

In the liquid layers, the non-dimensional displacements are related to the scalar and vector velocity potentials through the relations

$$u_L = \phi_{L,x} + \psi_{L,z}$$
$$w_L = \phi_{L,z} - \psi_{L,x} \qquad ...(16)$$

The governing equations in the fluid medium are given by

$$\left[\delta_L^2(1 + \varepsilon_L) + \frac{4}{3}\nu_L\frac{\partial}{\partial t}\right]\nabla^2\phi_L - \ddot{\phi}_L = 0 \qquad ...(17)$$

$$\nabla^2\psi_L - \frac{1}{\nu_L}\dot{\psi}_L = 0 \qquad ...(18)$$

$$T_L = \frac{-\varepsilon_L\rho_L\delta_L^2}{\bar{\beta}\rho}\nabla^2\phi_L \qquad ...(19)$$

where,

$$\delta_L^2 = \frac{c_L^2}{c_1^2},\ c_L^2 = \frac{\lambda_L}{\rho_L},\ \nu_L = \frac{\mu_L\omega^*}{\rho_L c_1^2},\ T'_L = \frac{T_L}{T_0}$$

$$\varepsilon_L = \frac{\beta_L^{*2}T_0^*}{\rho_L C_v^*\lambda_L},\ \bar{\beta} = \frac{\beta_L^*}{\beta_e},\ \beta_L^* = 3\lambda_L\alpha^* \quad ...(20)$$

Boundary Conditions

The boundary conditions at the solid-liquid interface $Z = 0$ to be satisfied are:

(*i*) The magnitude of the normal component of the stress tensor of the viscoelastic solid should be equal to the pressure of the liquid. This implies that

$$\frac{\ddot{\phi}}{\delta^2} - 2\left(1+\alpha_1\frac{\partial}{\partial t}\right)(\phi_{,xx} + \psi_{,xz})$$

$$= \frac{-\rho_L}{\rho\delta^2}\left(\ddot{\phi}_L - 2\nu_L(\dot{\phi}_{L,xx} + \dot{\phi}_{L,zz} + \dot{\psi}_{L,xx}\right) \quad ...(21)$$

(*ii*) The tangential components of stress tensor of viscoelastic solid and liquid should be equal, implying that

$$\frac{\ddot{\psi}}{\delta^2} - 2\left(1+\alpha_1\frac{\partial}{\partial t}\right)(\psi_{,xx} + \phi_{,xz})$$

$$= \frac{-\rho_L}{\rho\delta^2}\nu_L(2\dot{\phi}_{L,xz} + \dot{\psi}_{L,zz} + \dot{\psi}_{L,xx}) \quad ...(22)$$

(*iii*) The normal component of the displacement of the solid should be equal to that of the liquid. This leads to

$$\phi_{,z} - \psi_{,x} = \phi_{L,z} - \psi_{L,x} \quad ...(23)$$

(*iv*) The horizontal components of displacement of both the media should also be equal at the interface because of no slip condition. This provides us

$$\phi_{,x} + \psi_{,z} = \phi_{L,x} - \psi_{L,z} \quad ...(24)$$

(*v*) The thermal boundary condition is given by Noda et al. (2000)

$$T_{,z} + H(T - T_L) = 0 \quad ...(25)$$

where H is the Biot's constant. In the absence of liquid ($\rho_L = 0$, $\mu_L = 0$) that is for thermoelastic half-space, the boundary condition (25) corresponds to thermally insulated boundary for ($H \to 0$) and refers to isothermal one in case of ($H \to \infty$).

FORMAL SOLUTION

We assume solution of the form

$$f(x, z, t) = \bar{f}(z)\exp[i\xi(x - ct)] \quad ...(26)$$

where f is any one of the functions ϕ, ψ, T, ϕ_L, ψ_L and $c = \frac{\omega}{\xi}$ is the non-dimensional phase velocity, ω is the frequency and ξ is the wave number. Upon using solution (26) in equations (13-15) and (17-20), we get a system of differential equations which give the expressions for ϕ, ψ, T, ϕ_L, and ψ_L upon satisfying the radiation condition (Re (m_k^*, $\bar{\beta}^*$) ≥ 0, $k = 1, 2$) as

$$\phi = \sum_{k=1}^{2} A_k \exp\{-\xi m_k^* z + i\xi(x - ct\} \quad ...(27)$$

$$\psi = A_3 \exp\{-\xi\bar{\beta}^* z + i\xi(x - ct\} \quad ...(28)$$

$$T = -\frac{i\omega^{-1}\tau_1^{-1}\delta_0^*}{\beta_0^*}\sum_{k=1}^{2}\xi^2(\alpha^{*2} - m_k^{*2})$$

$$A_k \exp\{-\xi m_k^* z + i\xi(x - ct\} \quad ...(29)$$

For thermoelastic viscous solid half-space ($0 \leq z < \infty$) and

$$\phi_L = A_4 \sinh \xi\gamma_1(z + h)\exp\{i\xi(x - ct\} \quad ...(30)$$

$$\psi_L = A_5 \sinh \xi\gamma_2(z + h)\exp\{i\xi(x - ct\} \quad ...(31)$$

$$T_L = \xi^2 S_L A_4 \sinh \xi\gamma_1(z + h)\exp\{i\xi(x - ct\} \quad ...(32)$$

For viscous liquid boundary layer ($-h < z \leq 0$).

here

$$S_L = \frac{\varepsilon_L\rho_L\delta_L^2}{\bar{\beta}\rho\left[\delta_L^2(1+\varepsilon_L) - \frac{4}{3}i\omega\nu_L\right]}c^2,\quad \alpha^{*2} = 1 - \frac{i\omega^{-1}c^2}{\delta_0^*},$$

$$\bar{\beta}^{*2} = 1 - \frac{i\omega^{-1}c^2}{\alpha_1^*\delta^2},\quad \gamma_2^2 = 1 - \frac{i\omega^{-1}c^2}{\nu_L},$$

$$\gamma_1^2 = 1 - \frac{c^2}{\delta_L^2(1+\varepsilon_L) - \frac{4}{3}i\omega\nu_L},\quad m_k^{*2} = 1 - \frac{i\omega^{-1}}{\delta_0^*}a_k^2c^2,$$

$$k = 1, 2;\ \tau'_0 = t_0\,\delta_{1k} + i\omega^{-1}$$

$$\tau_0 = t_0 + i\,\omega^{-1},\ \tau_1 = t_1\delta_{2k} + i\,\omega^{-1},\ \alpha_0^* = \alpha_0 + i\omega^{-1},$$

$$\beta_0^* = \beta_0 + i\omega^{-1},\ \alpha_1^* = \alpha_1 + i\omega^{-1}$$

$$\delta_0^* = \delta_0 + i\omega^{-1},\ a_1^2 + a_2^2 = 1 - i\omega\delta_0^*\tau_0 + i\tau_1\tau_0^1\varepsilon_T\omega^3\beta_0^{*2},$$

$$a_1^2a_2^2 = \tau_0 \quad ...(33)$$

DERIVATION OF SECULAR EQUATION

Invoking the interface boundary conditions (21-25) to at the surfaces $z = 0$ and using expression of ϕ, ψ, T, ϕ_L, ψ_L from (27-32), we get a system of five equations in five unknowns (A_i, $i = 1, 2, 3, 4, 5$). The system of five linear equations so obtained will have non trivial solution if and only if the determinant of the coefficients vanishes. After algebraic reductions and simplifications of the determinant, the secular relation governing the motion of non-leaky Rayleigh waves is obtained as

$$\left\{\begin{array}{l}\left(m_1^{*2}+m_2^{*2}+m_1^*m_2^*-\alpha^{*2}\right)\left(R_0+aR_0^v\right)\\ \qquad -m_1^*m_2^*\left(m_1^*-m_2^*\right)\left(R_1+aR_1^v\right)\end{array}\right\}$$

$$=\frac{H}{\xi}\left\{\begin{array}{l}\left(m_1^*+m_2^*\right)\left(R_0+aR_0^v\right)-\left(a^{*2}+m_1^*m_2^*\right)\\ \left(R_1+aR_1^v\right)-\dfrac{i\omega\beta_0^*\tau_1 S_L\left(R_2+aR_2^v\right)}{\delta_0^*}\end{array}\right\} \qquad ...(34)$$

where

$$R_0 = p^{*2} + (\omega_L - p^*)p^*\ T_4\ T_5$$

$$R_0^v = -4\,p^* + 4a + (p^* + 2i\omega\alpha_1^*)\left(\gamma_2^2-1\right)\bar{\beta}^*\,T_5 + T_4\ T_5\ R_{01}^v$$

$$R_{01}^v = 2(-\ \gamma_1^2\ + 2)p^* + \left\{\left(p^*-\omega_L\right)+2a\left(\gamma_1^2-2\right)\right\}(\ \gamma_2^2\ + 1)$$

$$R_1 = -\ 4\,\bar{\beta}^*\,\omega^2\,\alpha_1^{*2} - (\omega_L\ +\ 2\ i\omega\alpha_1^*)\ 2i\omega\alpha_1^*\,\bar{\beta}^*\,T_4\ T_5$$

$$-\ \omega_L(P^*\ +\ 2i\omega\alpha_1^*)T_4$$

$$R_1^v = 4\,\bar{\beta}^*\,a\ +\ 8\ i\omega\alpha_1^*\,\bar{\beta}^*\ -2(1-\ \gamma_1^2)\ (P^*\ +\ 2i\omega\alpha_1^*)T_4$$

$$+\ \bar{\beta}^*\ \ T_4\ T_5\ \ R_{11}^v$$

$$R_{11}^v = 4i\omega\alpha_1^*(\ \gamma_1^2\ - 2)$$

$$-\ \left\{\left(\omega_L+2i\omega\alpha_1^*\right)-2a\left(\gamma_1^2-2\right)\right\}\ (\ \gamma_2^2\ + 1)$$

$$R_2 = T_4\ (p^*\ +\ 2i\omega\alpha_1^*)\ \{p^*\ +\ 2i\omega\alpha_1^*\,\bar{\beta}^*\,T_5\}$$

$$R_2^v = \ T_4\ (p^*\ +\ 2i\omega\alpha_1^*)\ \{(\ \gamma_2^2\ + 1)\,\bar{\beta}^*\,T_5 - 2\}$$

$$p^* = -\ 2i\omega\alpha_1^* - \frac{c^2}{\delta^2},\ T_4 = \frac{\tanh\xi\gamma_1 h}{\gamma_1},\ T_5 = \frac{\tanh\xi\gamma_2 h}{\gamma_2},$$

$$\omega_L = \frac{c^2\rho_L}{\rho\delta^2} \qquad ...(35)$$

This secular equation (34) contains complete information regarding phase velocity, attenuation coefficient and other characteristics of non-leaky Rayleigh waves. It is also noticed here that the secular dispersion relation for leaky Rayleigh waves in a thermo-viscoelastic solid bordered with viscous liquid half-space ($h \to \infty$) can be obtained from the relation (34) by replacing $\dfrac{\tanh\xi\gamma_k h}{\gamma_k} \to \dfrac{1}{\gamma_k}$, ($k$ = 1, 2) respectively.

PARTICULAR CASES OF DISPERSION RELATION

This section is devoted to the reduction of the secular equation (34) under different situations such as

(*i*) Coupled Thermoelasticity. In the absence of the viscoelastic effect of solid, the mechanical relaxation times $\alpha_0^* = 0 = \alpha_1$ and hence $\alpha_0^* = \alpha_1^* = i\omega^{-1}$, $\delta_0^* = i\omega^{-1} = \beta_0^*$ and consequently, we have

$$\alpha^{*2} = (1-c^2) = \alpha^2,\ \bar{\beta}^{*2} = \left(1-\frac{c_2}{\delta^2}\right) = \beta^2\ ,$$

$$m_k^{*2} = (1-\ a_k^2\,c^2) = a_k^2\,,\ k = (1,\ 2).$$

Then the above secular equation (34) reduces to that of equation Sharma et al. (2008AMM). Further deductions are followed in the similar way and discussed in detail there.

(*ii*) Viscoelasticity. In case of uncoupled thermoelasticity, the thermal and mechanical fields get uncoupled and remain independent of each other $\varepsilon_T = 0 = \varepsilon_L$. Moreover, no heat transference will take place leading to H = 0. Consequently, the secular equation (34) reduces to

$$\left(R_0+aR_0^v\right)\ -\ m_1^*\left(R_1+aR_1^v\right)\ = 0 \qquad ...(36)$$

where the expressions of R_0, R_0^v, R_1, R_1^v are given by equation (35).

(*iii*) Elasticity. On setting $\alpha_0 = 0 = \alpha_1$ in equation, we obtain the transcendental Rayleigh frequency equations of wave propagation, which further reduces to that of Graff (1991) in the absence of fluid.

SOLUTION OF THE SECULAR EQUATION

The characteristic roots m_k^*, (k = 1, 2) and γ_2 defined by equations are complex and therefore, the wave number and phase velocity of the waves are complex quantities. Therefore, the waves get attenuated in space.

If we write

$$c^{-1} = V^{-1} + i\ \omega^{-1}\ Q \qquad ...(37)$$

such that $\xi = R + iQ$, $R = \dfrac{\omega}{V}$, where V and Q are real. The exponent in the plane wave solution becomes $\boldsymbol{iR}\,(x - Vt) - Qx$ meaning thereby V is the propagation speed and Q the attenuation coefficient of the waves. Substituting equation (37) in secular equation (34) and other relevant relations, the phase velocity (V), attenuation coefficient (Q) and specific loss factor of energy dissipation can be computed at different values of (R) for the propagation of non-leaky Rayleigh waves by functional iteration method outlined below.

The secular equation (34) is of the form $F(c) = 0$ which upon using representation (37) leads to a system of two real equations $f(V, Q) = 0$ and $g(V, Q) = 0$. In order to apply functional iteration method we write $V = f^*(V, Q)$ and $Q = g^*(V, Q)$, where the functions f^* and g^* are selected in such a way that they satisfy the conditions

$$\left|\frac{\delta f^*}{\partial V}\right| + \left|\frac{\delta f^*}{\partial Q}\right| < 1, \quad \left|\frac{\delta g^*}{\partial V}\right| + \left|\frac{\delta g^*}{\partial Q}\right| < 1 \qquad ...(38)$$

For all V, Q in the neighborhood of the root. If (V_0, Q_0) be the initial approximation to the root, then we construct the successive approximations according to the formulae

$$\begin{aligned} V_1 &= f^*(V_0, Q_0), \quad Q_1 = g^*(V_1, Q_0) \\ V_2 &= f^*(V_1, Q_1), \quad Q_2 = g^*(V_2, Q_1) \\ &\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ V_{n+1} &= f^*(V_n, Q_n), \quad Q_{n+1} = g^*(V_{n+1}, Q_n) \end{aligned} \qquad ...(39)$$

The sequence $\{V_n, Q_n\}$ of approximations to the root will converge to the actual value (V_0, Q_0) of the root provided (V_0, Q_0) lies in the neighbourhood of the actual root. For the initial value $c = c_0 = (V_0, Q_0)$, the roots m_i^* are computed from equation for each value of wave number R, for assigned frequency. The values of m_i^* $(i = 1, 2)$ so obtained are then used in secular equation (34) to obtain the current values of V and Q each time which are further used to generate the sequence (39). The process is terminated as and when the condition $|V_{n+1} - V_n| < \varepsilon$, ε being arbitrarily small number to be selected at random to achieve the accuracy level, is satisfied. The procedure is continuously repeated for different values of the wave number R to obtain corresponding values of the phase velocity (V) and attenuation coefficient (Q). The specific loss of energy (SL) is given by (40) has also been computed.

Specific Loss

The ratio of energy dissipated (ΔW) in a specimen through a stress cycle, to the elastic energy (W) stored in the specimen when the strain is maximum, is called 'specific loss'. According to Kolsky (1963) in case of sinusoidal plane wave of small amplitude, the specific loss $\left(\frac{\Delta W}{W}\right)$ equals to 4π times the absolute value of imaginary part of ξ to the real part of ξ *i.e.* Hence here it is given by

$$\left|\frac{\Delta W}{W}\right| = 4\pi \left|\frac{VQ}{\omega}\right| \qquad ...(40)$$

NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the theoretical results obtained in the previous section, some numerical results are presented. The material chosen for this purpose is copper, the physical data for which is given in Table 1 below. The values of non-dimensional thermal relaxation parameters ($\bar{t}_0 = t_0\omega^*$, $\bar{t}_1 = t_0\omega^*$) are taken as $t_1 = 0.3$, $t_0 = 0.5$. The value of Biot's constant is taken as $H = 0.1$ for numerical calculations. The liquids chosen for the purpose of numerical calculations are light and heavy water, for which the velocity of sound and density are given by $c_L = 1.5 \times 10^3$ m/s, $\rho_L = 1000$ Kg m^{-3} respectively. Upon using representation (37), the complex secular equation (34) is solved by using functional iteration method to obtain phase speed (V) and attenuation coefficient (Q) after computing complex characteristic roots.

Table 1 : Physical data for Copper (Sharma, 40).

S. No.	Quantity	Units	Numerical Value
1.	ρ	Kg m^{-3}	8.950×10^3
2.	K	Cal m^{-1} s^{-1} K^{-1}	1.13×10^2
3.	ε_T	—	0.00265
4.	λ_e	Nm^{-2}	8.2×10^{10}
5.	μ_e	Nm^{-2}	4.2×10^{10}
6.	ω^*	sec^{-1}	4.347×10^{13}
7.	T_0	K	300
8.	α_T	K^{-1}	1.0×10^{-8}
9.	$\alpha_0 = \alpha_1$	sec	6.8831×10^{-8}

The specific loss factor of energy dissipation has also been computed from equation (38). The computed simulated results in respect of various wave characteristics in the non-dimensional form corresponding to three situations of liquid loading namely, inviscid (ideal) liquid ($\rho_L \neq 0.0$, $\mu_L = 0.0$), low viscous liquid ($\rho_L \neq 0.0$, $\mu_L = 0.1$) and high viscous liquid ($\rho_L \neq 0.0$, $\mu_L = 1.0$) have been plotted graphically in Figs.1 to 9 in case of *GL*-theory.

Fig.1 represents the variations of phase velocity profiles (V) with respect to wave number (R) on linear-log scales for ideal (inviscid), low viscous and high viscous types of fluid loading. Phase velocity profiles show decreasing trend in the wave number range $0 \leq R \leq 1.2$ in all the three considered cases of liquid loading with slight difference in their magnitudes before these become steady and stable afterwards. The variations of attenuation coefficient (Q) versus with wave number (R) have been plotted in Fig.2 in case of inviscid/viscous fluid loadings and free surface on linear-log scales. It is noticed that the profiles of this quantity in all the considered conditions prevailing at the interfacial surface increase monotonically in the wave number range $0 \leq R \leq 1$ and become steady and stable in case of free surface condition in contrast to fluid loaded one in which the profiles follow stable trends after observing fluctuating behavior for $R \geq 1$. It is also observed that the effect of inviscid fluid on this quantity is approximated between free surface and viscous fluid loading conditions.

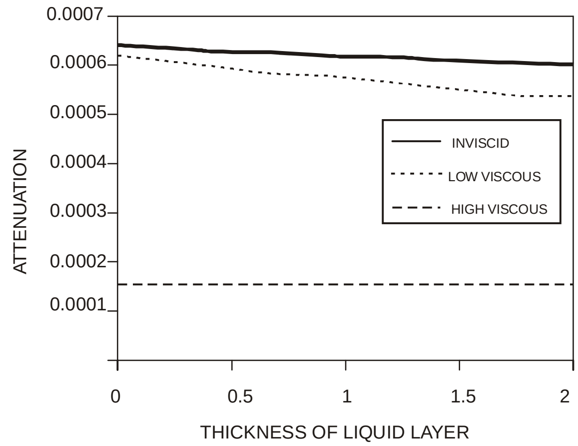

Fig.1. Variation of phase velocity with wave number.

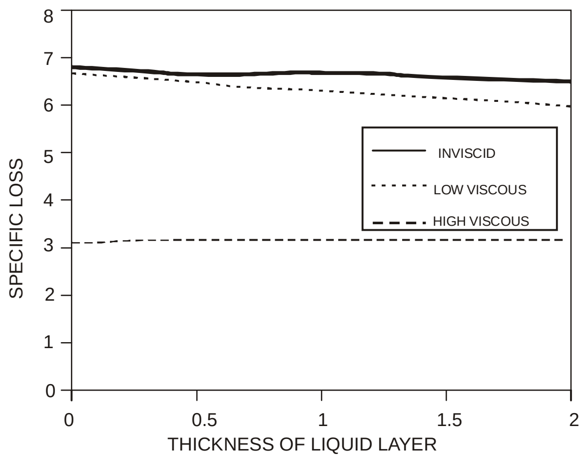

Fig.2. Variation of attenuation with wave number.

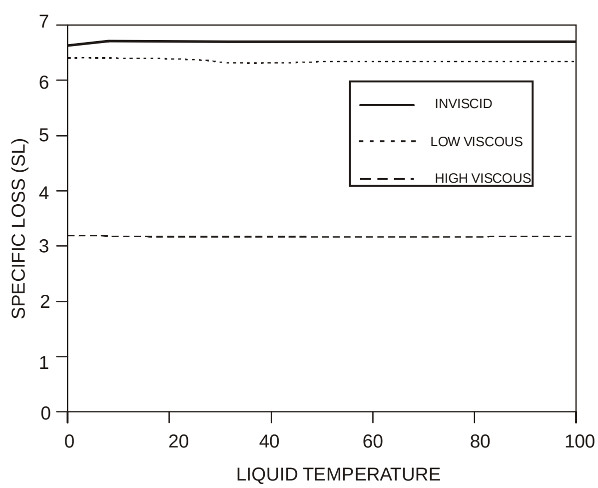

Fig.3. Variation of specific loss with wave number.

Fig.3 shows the variations of specific loss (SL) versus wave number (R) under free and inviscid/viscous fluid loading conditions. The profiles of this quantity corresponding to free and high viscous fluid loading conditions are more or less dispersionless at all wavelengths except in the wave number range $0 \le R \le 1$. However, the profiles of specific loss (SL) pertaining to inviscid and low viscous liquid loading at the interface follow dispersive behaviour. The degree of dispersion in case of inviscid fluid loading is quite high as compared to rest of the cases of loadings, though it also becomes steady and stable alike other profiles at short wavelengths. The variations of phase velocity (V) with ambient liquid temperature (T_0^*) are plotted in Fig.4. It is noticed that the phase velocity profiles in all the three considered cases of liquid loading have similar trend and behavior with liquid loading temperature except small differences in their magnitude. The magnitude in case of high viscous liquid loading is noticed to be nearly the double to that of inviscid/low viscous fluid. All profiles are dispersionless and follow nearly linear variations with temperature change of liquid.

The variations of attenuation coefficient (Q) versus ambient liquid temperature (T_0^*) have been presented in Fig.5. It is revealed that profiles of this quantity pertaining to inviscid, low viscous and high viscous fluid loadings exhibit linear behaviour with increasing ambient temperature of the fluid and hence more or less dispersionless throughout the considered range of temperature. However, the magnitude of this quantity due to increasing viscosity is approximately one-third to that of inviscid fluid loading. Similar behaviour and trends of variations of various profiles of specific loss factor (SL) of energy dissipation have also been observed from Fig.6 except the magnitude of (SL) factor is approximately ten-thousand time to that of attenuation coefficient (Q) in Fig.5.

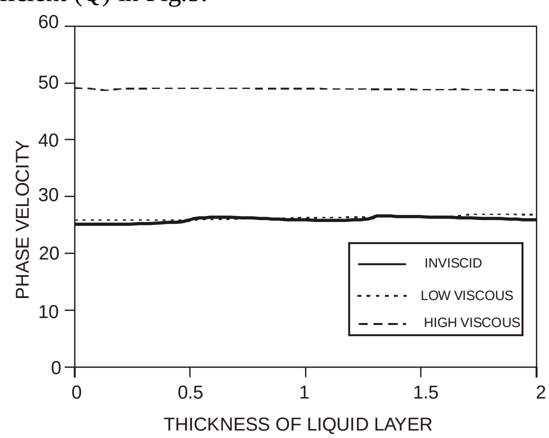

Fig.4. Variation of phase velocity with liquid temperature.

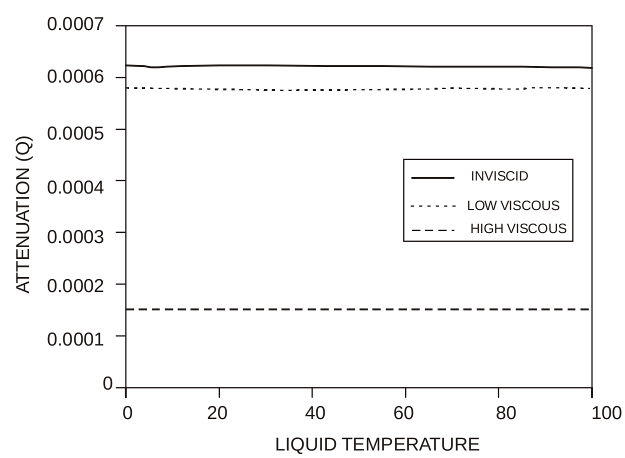

Fig.5. Variation of attenuation with liquid temperature.

It is also noticed while the magnitude of phase velocity is quite large in case of high viscous liquid loading as compared to that of inviscid/low viscous one at all values of the ambient liquid temperature (T_0^*) whereas the trend get reversed in case of attenuation coefficient (Q) and specific loss factor (SL). The variations of phase velocity (V), attenuation coefficient (Q) and specific loss factor (SL) versus liquid layer thickness (h) are plotted in Figs. 7 to 9. The profiles of all these quantities follow more or less linear variations and thereby exhibit dispersionless behaviour with increasing liquid layer thickness with small

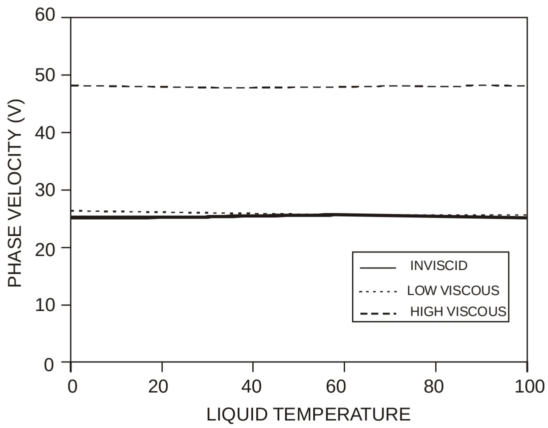

Fig.6. Variation of specific loss with liquid temperature.

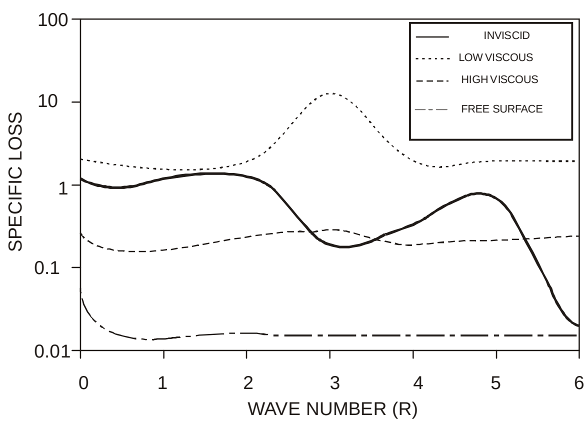

Fig.7. Variation of phase velocity with thickness of the liquid layer.

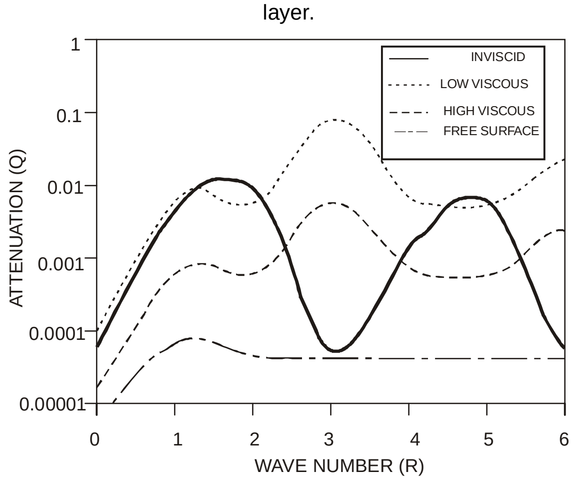

Fig.8. Variation of attenuation with thickness of the liquid layer.

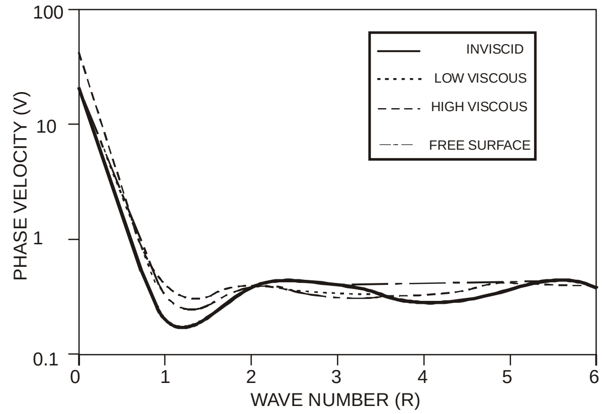

Fig.9. Variation of specific loss with thickness of the liquid layer.

deviations in respect of attenuation and specific loss profiles at large values of the liquid thickness (h). Magnitude of attenuation and specific loss factor is quite small in case of high viscous fluid as compared to that of inviscid/low viscous fluid profiles in contrast to that of phase velocity in which this trend gets reversed.

CONCLUSIONS

Significant changes in the profiles of phase velocity, attenuation coefficient and specific loss factor of energy dissipation due to mechanical relaxation times have been observed in addition to the effects of different cases of considered fluid loadings. The behaviour of profiles pertaining to inviscid fluid loading has been observed to be approximated between those of free surface and viscous fluid loading profiles. The viscosity of both media results in decreasing the magnitude of attenuation and specific loss factor due to increasing liquid layer ambient temperature as well as thickness whereas it contributes in increasing the phase velocity of the interfacial waves. The liquid overlying the thermoviscoelastic solid halfspace has been successfully modelled as normal load (hydrostatic pressure) and thermal source simultaneously.

REFERENCES

[1] Bland, D.R., The theory of linear viscoelasticity, Pergamon Press, Oxford (1960).

[2] Gurtin, M.E. and Sternburg, E., On the linear theory of viscoelasticity, *Arch. Rat. Mech. Anal.*, **11:** 291-356(1962).

[3] Lord, H.W. and Shulman, Y., A generalized dynamical theory of thermo elasticity, *J. Mech. Phys. Solids*, **15:** 299-309(1967).

[4] Green, A.E. and Lindsay, K.A., Thermoelasticity, *J. Elasticity*, **2:** 1-7(1972).

[5] Green, A.E. and Laws, N., On the entropy production inequality. *Arch. Rat. Mech. Anal.* **45:** 47-53(1972).

[6] Ackerman, C.C., Bertman, B., Fairbank, H.A., and Guyer R. A., Second sound in solid Helium, *Phys. Rev. Lett.*, **16:** 789-791(1966).

[7] Ackerman, C.C. and Overton, W.C., 1969: Second sound in solid Helium-3, *J. Phys. Rev. Lett.*, **22:** 764-766.

[8] Guyer, R.A. and Krurmhansal, J.A., Thermal conductivity, second sound and phonon hydrodynamic phenomena in non-metallic crystals, *Phys. Rev.*, **148:** 778-788(1966).

[9] Green, A.E., A note on linear thermo elasticity, *Mathematika*, **19:** 69-75(1972).

[10] Rayleigh, L., On waves propagated along the plane surface of an elastic solid, *Proc. London Math. Soc.*, **17:** 4-11(1885).

[11] Love, A.E.H, Some problems of Geodynamics. *Cambridge University Press London* (1911), (1926).

[12] Ewing, W.M., Jardetszky, W.S., Press, F., Elastic waves in layered media. *McGraw Hill New York* (1957).

[13] Andle, J.C. and Vetelino, J.F., Acoustic wave biosensors. *Sens. and Actuat.*, A, **44:** 167-176(1994).

[14] Kovacs, G., Vellekoop, M.J., Haueis, R.G.W. and Venema, A., A Love wave sensor for (bio) chemical sensing in liquids, *Sens. and Actuat.* A, **43:** 38-43 (1994).

[15] Kurtze, G. and Bolt, R.H., On the interaction between plate bending wave and radiation load, *Acoustica*, **9:** 238-242(1959).

[16] Plona, T.J., Behravesh, M. and Mayer, W.G., Rayleigh and Lamb waves at liquid-solid boundaries, *Ultrasonics*, **13:** 171-174(1975).

[17] Wu, J.R. and Zhu, Z.M., The propagation of Lamb waves in a plate bordered with layers of a liquid, *J. Acoust. Soc. Am.*, **91:** 861-867(1991).

[18] Banerjee, S. and Kundu, T., Ultrasonic field modeling in plates immersed in fluid, *Int. J. Solids Structs.*, **44:** 6013-6029(2007).

[19] Bossart, R., Joly, N. and Bruneau, M., Hybrid numerical and analytical solutions for acoustic boundary problems in thermo-viscous fluids, *J. Sound and Vib.*, **263:** 69-84(2003).

[20] Craster, R.V. and Williams, D.P., A reciprocity relation for fluid-loaded elastic plates that contain rigid defects, *J. Sound and Vib.*, **235:** 655-670(2000).

[21] Wang, H. and Zhang, H.S., Boundary element method for simulating the coupled motion of a fluid and a three-dimensional body, *Appl. Math. and Comput.*, **190:** 1328-1343(2007).

[22] Skelton, E.A., Line force receptance of anelastic cylindrical shell with heavy exterior fluid loading, *J. Sound and Vib.*, **256:** 131-153(2002).

[23] Krishna, B.V. and Ganesan, N., Studies on fluid-filled and submerged cylindrical shells with constrained viscoelastic layer, *J. Sound and Vib.*, **303:** 575-595(2007).

[24] Nayfeh, A.H. and Nagy, P.B., Excess attenuation of leaky Lamb waves due to viscous fluid loading, *J. Acoust. Soc. Am.*, **101:** 2649-2658(1997).

[25] Zhu, Z. and Wu, J., The propagation of Lamb waves in a plate bordered with viscous liquid, *J. Acoust. Soc. Am.*, **98:** 1057-1064(1995).

[26] Wu, T.T. and Wu, T.Y., Surface waves in coated anisotropic medium loaded with viscous liquid, ASME, *J. Appl. Mech.*, **67:** 262-266(2000).

[27] Hassan, W. and Nagy, P.B., On the anomalously low attenuation of the leaky Rayleigh wave in a fluid-filled cylindrical cavity, *J. Acoust. Soc. Am.*, **104:** 1246-1255(1998).

[28] Garadzhaev, A., A note on the spectral theory of problems on normal oscillation of an ideal compressible fluid in rotating elastic shell, *Appl. Math. and Comput.*, **191:** 302-307(2007).

[29] Venkatesan, M. and Ponnusammy, P., Wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section immersed in a fluid, *Int. J. Mechanical Sci.*, **49:** 741-751(2007).

[30] Qi, Q., Attenuated leaky Lamb waves, *J. Acoust. Soc. Am.*, **95:** 3222-3231(1994).

[31] Cho, Y. and Rose, J.L., Guided waves in water loaded hollow cylinder, *Non Dest. Testing and Eval.*, **12:** 323-339(1996).

[32] Popovics, J.S., Zhu, J. and Schubert, F., Leaky Rayleigh and Scholte waves at the fluid-solid interface subjected to transient point loading, *J. Acoust. Soc. Am.*, **116:** 2101-2110(2004).

[33] Cherednichenko, Kirill D., On propagation of attenuated Rayleigh waves along a fluid-solid interface of arbitrary shape, *Q. J. Mech. Appl. Math.*, **59**, 75-94(2006).

[34] Mozhaev, V.G. and Weihnacht, M., Subsonic leaky Rayleigh waves at liquid-solid interfaces, *Ultrasonics*, **40:** 927-933(2002).

[35] Sharma, J.N., Kumar, S. and Sharma, Y.D., Propagation characteristics of Rayleigh surface waves in microstretch thermoelastic continua under inviscid fluid loadings, *J. Thermal Stresses*, **31:** 18-39(2008).

[36] Hasheminejad, S.M. and Safari, N., Acoustic scattering from viscoelastically coated spheres and cylinder in viscous fluids, *J. Sound and Vib.*, **280:** 101-125(2005).

[37] Shuvalov, A.L., Poncelet, O. and Deschamps, M., Analysis of the dispersion spectrum of fluid-loaded anisotropic plates: flexural-type branches and real-

valued loops, *J. Sound and Vib.*, **290:** 1175-1201(2006).

[38] Zhao, Y., Shen, Z., Lu, J. and Ni, Xi., Simulation of leaky Rayleigh wave at air-solid cylindrical interfaces by finite element method, *Ultrasonics*, **44:** Supplement 1, e1169-e1172(2006).

[39] Ghattas, O. and Li, Xi., A variational finite element method for stationary nonlinear fluid-solid interaction, *J. Comput. Phys.*, **121:** 347-356(1995).

[40] Sharma, J.N., Some considerations on the Rayleigh-Lamb wave propagation in visco-thermoelastic plates, *J. Vib. and Cont.*, **11:** 1311-1335(2005).

[41] Sharma, J.N. and Pathania, V., Thermoelastic waves in a plate bordered with layers of inviscid liquid, *J. Thermal Stresses,* **26:** 149-166(2003).

[42] Sharma, J.N. and Pathania, V., Crested waves in thermoelastic plates immersed in liquid, *J. Vib. and Cont.*, **11:** 347-370(2005).

[43] Sharma, J. N. and Pathania, V., Propagation of leaky surface waves in thermoelastic solids due to inviscid fluid loadings, *J. Thermal Stresses,* **28:** 485-519(2005).

[44] Ugurlu, B., Kutlu, A., Ergin, A., Omurtag, M.H., Dynamics of a rectangular plate resting on an elastic foundation and partially in contact with a quiescent fluid, *J. Sound and Vib.*, **317:** 308-328(2008).

[45] Sorokin, S.V. and Chubinskij, A.V., On the role of fluid viscosity in wave propagation in elastic plates under heavy fluid loading, *J. Sound and Vib.*, **311:** 1020-1038(2008).

[46] Sharma, J.N., Pathania, V. and Gupta, S.K., Thermoelastic Lamb waves in a transversely isotropic plate bordered with layers of inviscid liquid *Int. J. Engng. Sci.*, **41:** 1219-1237(2003).

[47] Sharma, J.N., Chand, R. and Othman, M.I.A., On the propagation of Lamb waves in visco-thermoelastic plates under fluid loadings, *Int. J. Engng. Sci.*, **47:** 391-404(2009).

[48] Sharma, J.N. and Pathania, V., Generalized thermoelastic waves in anisotropic plates sandwiched between liquid layers, *J. Sound and Vib.*, **278:** 383-411(2004).

[49] Kaliski, S., Absorption of magneto-viscoelastic surface waves in a real conductor in magnetic field, *Proc. Vib. Probs.*, **4:** 319-329(1963).

[50] Achenbach, D., Wave propagation in elastic solids, North-Holland America (1973).

[51] Graff, K.F., Wave motion in elastic solids, Dover Publications, New York (1991).